

LATTICE GAUGE THEORY

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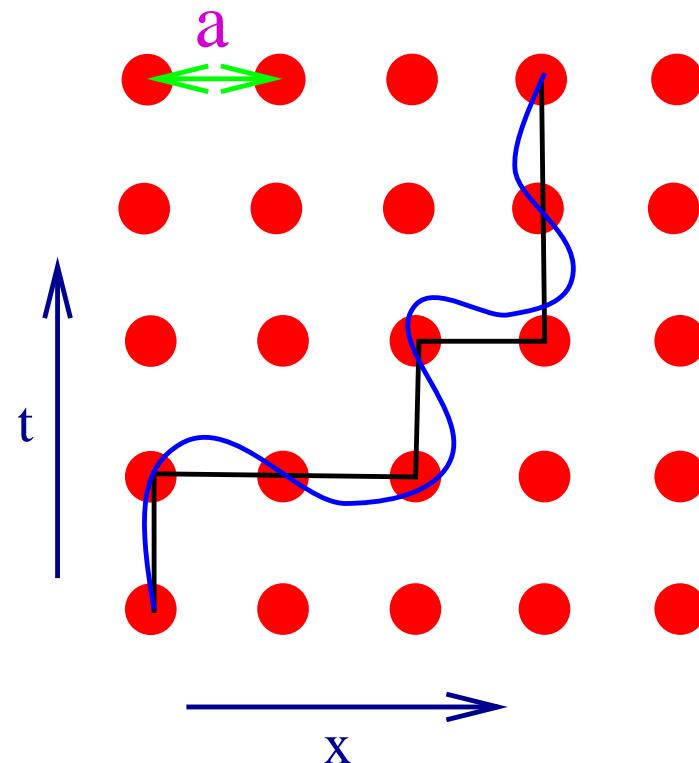
Why the lattice?

What drove us to it?

Space-time Lattice

A mathematical trick

World lines \rightarrow discrete hops



Lattice spacing a

$a \rightarrow 0$ for physics

$a = \text{cutoff} = \pi/\Lambda$

Field theory has divergences

- bare charge, mass infinite
- must “regulate” for calculation
- Pauli Villars, dimensional regularization
- based on Feynman diagrams

But important non-perturbative effects

- confinement
- chiral symmetry breaking

need a “non-perturbative” regulator

Wilson’s strong coupling lattice theory (1973)

- strong coupling limit confines
- only hadrons can move

space-time lattice = non-perturbative cutoff

Lattice gauge theory

- A mathematical trick
- Minimum wavelength = lattice spacing a
- Maximum momentum = π/a
- Allows computations
- Defines a field theory

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- Be discrete, do it on the lattice
 - Be indiscreet, do it continuously

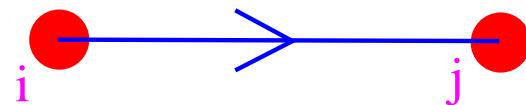
Wilson's formulation

local symmetry + theory of phases

Variables:

- Gauge fields generalize “phases”

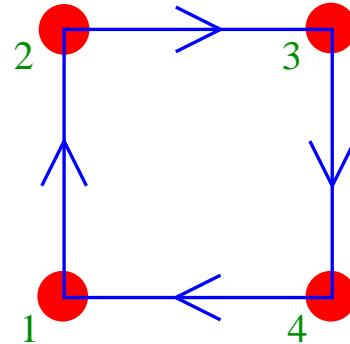
$$U_{i,j} \sim \exp(i \int_{x_i}^{x_j} A^\mu dx_\mu)$$



- On links connecting nearest neighbors
- $U_{i,j}$ = 3 by 3 unitary matrix $\in \text{SU}(3)$
- 3 quarks in a proton

Dynamics:

- Sum over elementary squares, “plaquettes”



$$U_p = U_{1,2}U_{2,3}U_{3,4}U_{4,1}$$

- like a “curl”
- flux through corresponding plaquette.

$$S = \int d^4x F^{\mu\nu} F_{\mu\nu} \rightarrow \sum_p \left(1 - \frac{1}{3} \text{ReTr} U_p \right)$$

Quantum mechanics:

- via path integral
- sum over paths —→ sum over phases

$$Z = \int (dU) e^{-\beta S}$$

- invariant group measure
- β defines the “bare” charge

$$\beta = \frac{6}{g_0^2}$$

- must renormalize as $a \rightarrow 0$

Parameters

$$a \rightarrow 0$$

Asymptotic freedom:

$$g_0^2 \rightarrow 0$$

Overall scale from “dimensional transmutation”

- Coleman and Weinberg

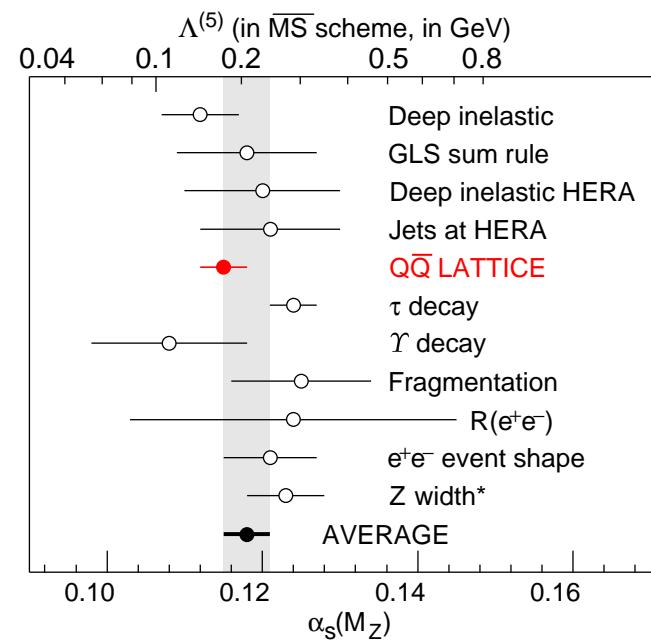
Only the quark masses!

$m_q = 0$: parameter free theory

- $m_\pi = 0$
- m_ρ/m_p determined
- close to reality

Example: strong coupling determined

$$\alpha_s(M_Z) = 0.115 \pm 0.003$$



(PDG, 1999)

(charmonium spectrum for input)

Numerical Simulation

$$Z = \int dU e^{-\beta S}$$

10^4 lattice \Rightarrow

- $10^4 \times 4 \times 8 = 320,000$ dimensional integral
- 2 points/dimension \Rightarrow

$$2^{320,000} = 3.8 \times 10^{96,329} \quad \text{terms}$$

- age of universe $\sim 10^{27}$ nanoseconds

Use statistical methods

- $Z \longleftrightarrow$ partition function
- $\frac{1}{\beta} \longleftrightarrow$ temperature

Find “typical” equilibrium configurations C

$$P(C) \sim e^{-\beta S(C)}$$

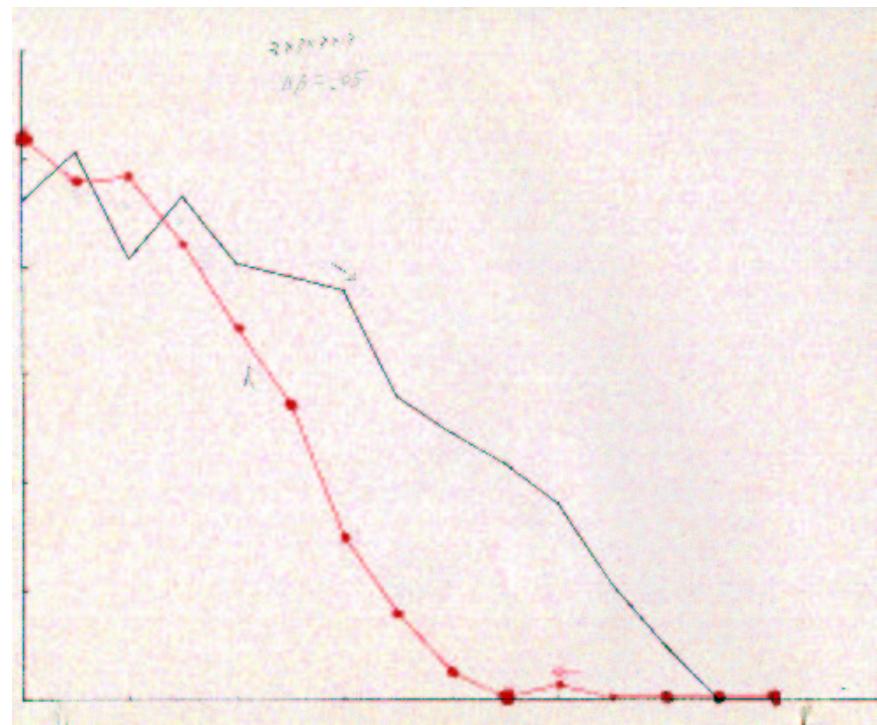
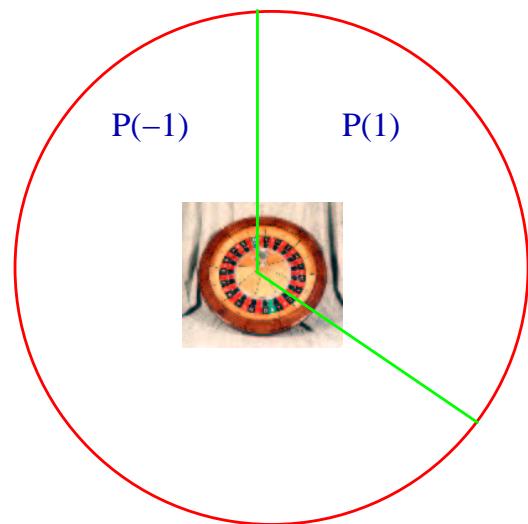
Use a Markov process

$$C \rightarrow C' \rightarrow \dots$$

Z_2 example: (L. Jacobs, C. Rebbi, MC)

$$U = \pm 1$$

$$P(1) = \frac{e^{-\beta S(1)}}{e^{-\beta S(1)} + e^{-\beta S(-1)}}$$

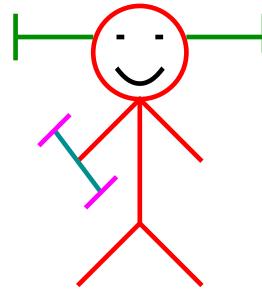


Monte Carlo methods

Make random field changes biased by Boltzmann weight.
Converge towards configurations in “thermal equilibrium.”

$$P(C) \sim e^{\beta S}$$

In principle can measure anything.
Fluctuations → theorists have error bars!

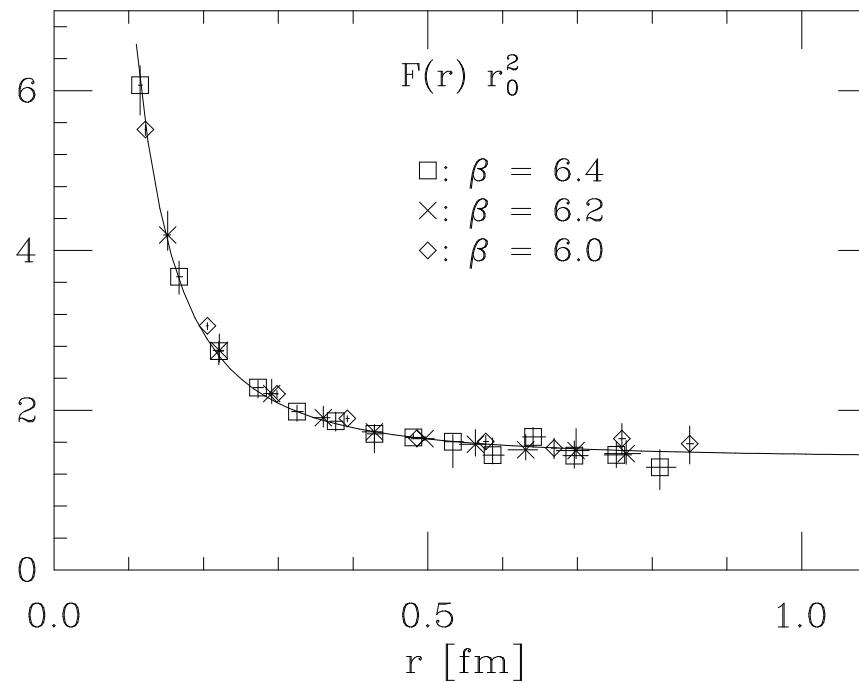


Systematic errors:

- finite volume
- finite lattice spacing
- quark mass extrapolations
- valence approximation for quarks

Interquark force

- constant at large distance
- confinement



C. Michael, hep-lat/9509090

Quarks: serious unsolved problems

Anticommuting fields

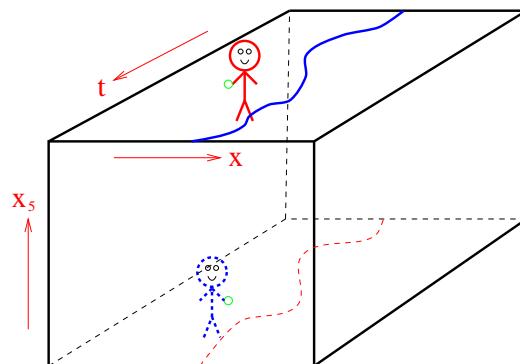
- $\not\rightarrow$ classical statistical mechanics
- Integrate out as a determinant
- Tedious to simulate.

Chemical potential background baryon density

- Non-positive weight.
- No viable algorithms known!

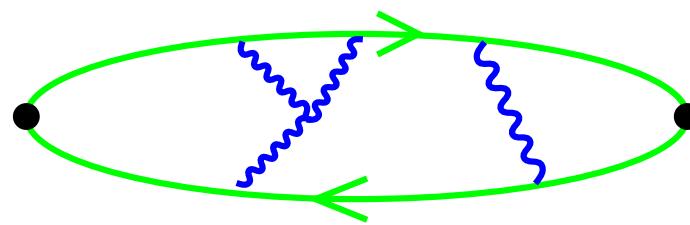
Chiral fermions and the “standard model”

- Unsolved difficulties tied with anomalies.
- Lots of recent activity.
- My favorite: 4d world an interface in 5d

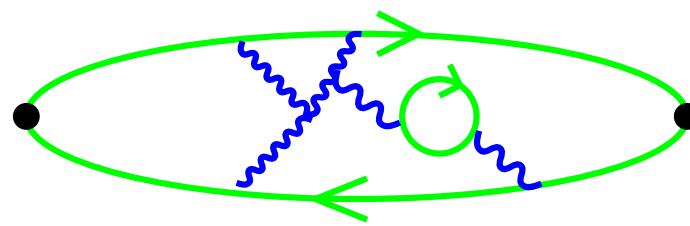


Valence or Quenched approximation:

- Simulate gauge fields ignoring $|D + m|$
- Propagate quarks in background gauge field
- include:



- neglect:



Saves orders of magnitude in computer time

Singular in the light quark limit

Hadronic spectra: as $t \rightarrow \infty$

$$\langle \phi(t) \phi(0) \rangle \longrightarrow e^{-mt}$$

- m = mass of lightest hadron created by ϕ
- Bare quark mass is a parameter

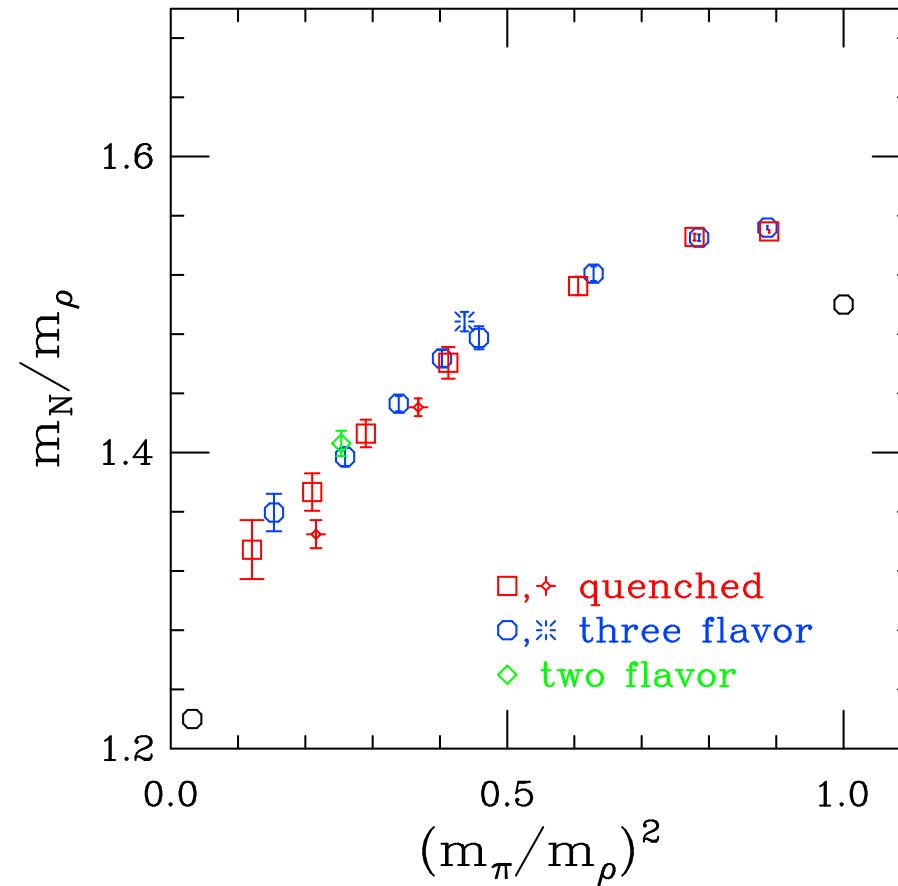
Chiral symmetry:

$$m_\pi^2 \sim m_q$$

Adjust M_q to get m_π/m_ρ (M_s for the kaon)

all other mass ratios determined

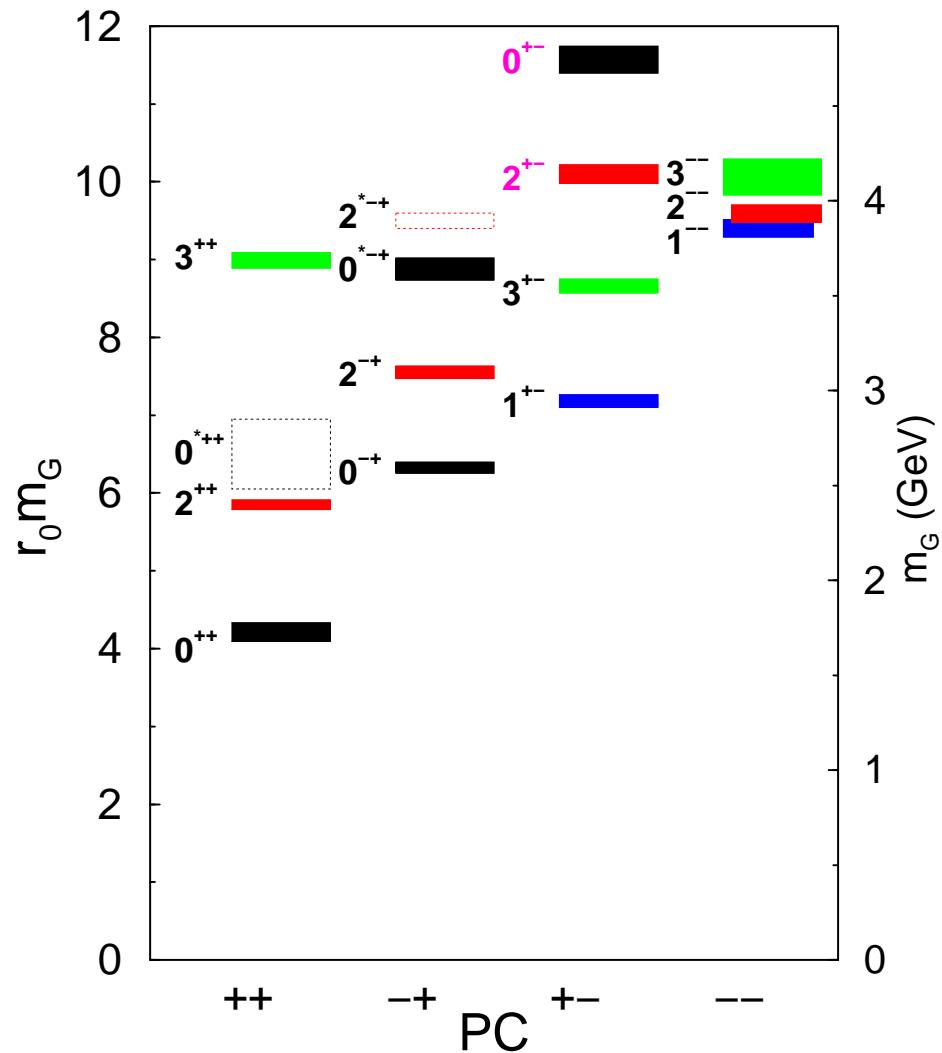
“APE” or “Edinburgh” plot:



- improved Kogut-Susskind quarks, $16^3 \times 48$ lattice
- MILC collaboration, Phys. Rev. D 64, 054506 (2001)

Glueballs

- gluonic excitations
- no quarks



- Morningstar and Peardon, Phys. Rev. D 60, 034509 (1999)
- quenched, anisotropic lattice

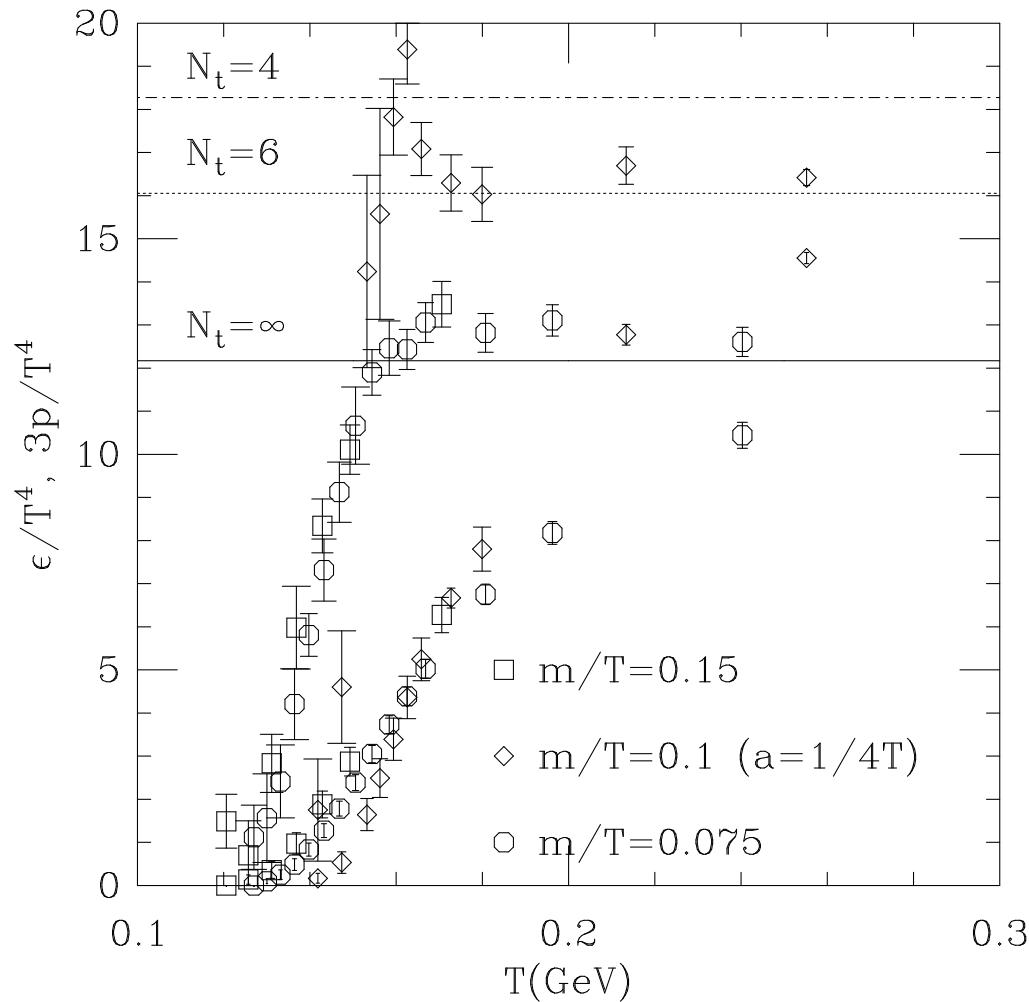
Quark Gluon Plasma

Finite temporal box of length t

$$Z \sim \text{Tr } e^{-Ht}$$

- $1/t \leftrightarrow$ temperature
- confinement lost at high temperature
- chiral symmetry manifestly restored
- $T_c \sim 235$ MeV, 0 flavors (quenched)
- $T_c \sim 150$ MeV, 2 flavors

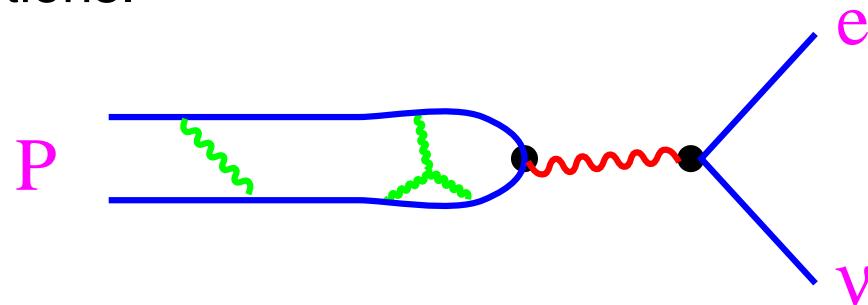
Energy ϵ and pressure p versus temperature.



Bernard *et al.*, MILC collaboration, Dec. 1996

Matrix elements

To test standard model predictions for weak decays, need strong interaction corrections.



G. Martinelli, Lattice 2001:

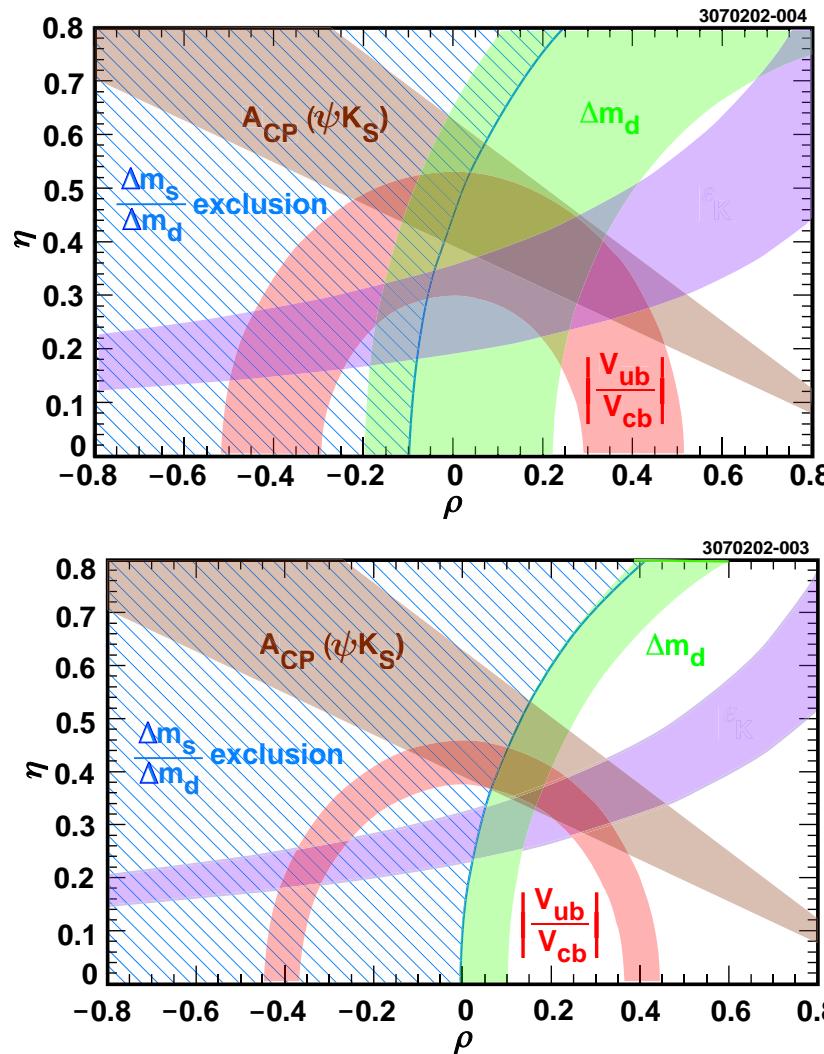
Table 2

Lattice results for $\Delta I = 1/2$ transitions using $K \rightarrow \pi$ matrix elements from RBC and CP-PACS. The experimental numbers are also given.

Reference	$Re\mathcal{A}_0$	$Re\mathcal{A}_2$	$Re\mathcal{A}_0/Re\mathcal{A}_2$	ϵ'/ϵ
CP-PACS [3]	$16 \div 2 \times 10^{-8}$	$1.3 \div 1.5 \times 10^{-8}$	$9 \div 12$	$-7 \div -2 \times 10^{-4}$
RBC [4]	$29 \div 13 \times 10^{-8}$	$1.1 \div 1.2 \times 10^{-8}$	$24 \div 27$	$-8 \div -4 \times 10^{-4}$
Exps. [16–18] 2001	33.3×10^{-8}	1.5×10^{-8}	22.2	$17.2 \pm 1.8 \times 10^{-4}$

- $\Delta I = 1/2$ rule verified
- ϵ'/ϵ puzzling?
- quenching errors under investigation; seem to be large
- heavy use of chiral perturbation theory

Constraints on the CKM Matrix



Constraints on the Standard Model parameters ρ and η (one sigma confidence level). For the Standard Model to be correct, they must be restricted to the region of overlap of the solidly colored bands. The figure on the top shows the constraints as they exist today. The figure on the bottom shows the constraints as they would exist with no improvement in the experimental errors, but with lattice gauge theory uncertainties reduced to 3%. R. Patterson, Cornell University.

The Lattice SciDAC Project

66 US lattice theorists; 9 member executive committee:

R. Brower, (Boston U.) N. Christ (Columbia U.), M. Creutz (BNL), P. Mackenzie (Fermilab), J. Negele (MIT), C. Rebbi (Boston U.), S. Sharpe (U. Washington), R. Sugar (UCSB) and W. Watson, III (JLab)

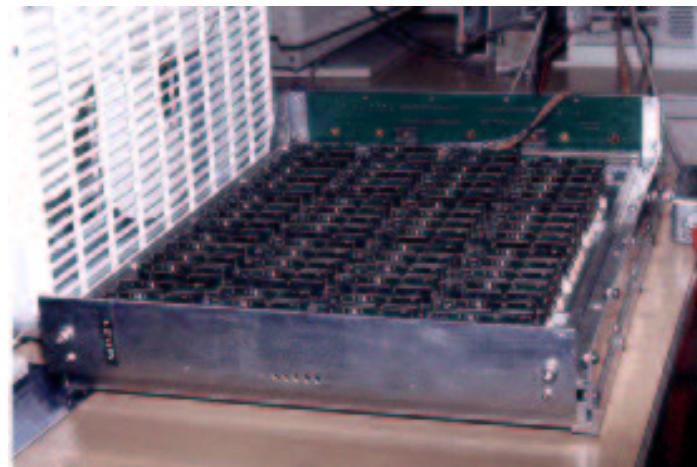
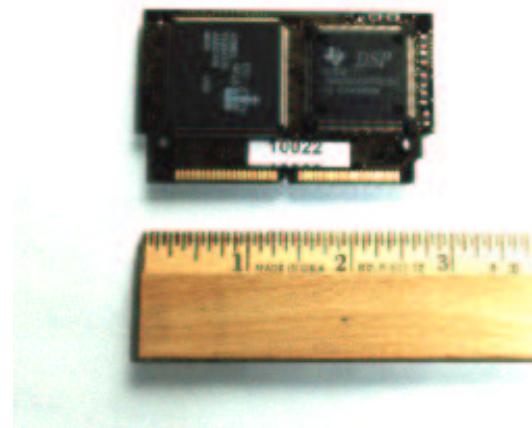
Two prong approach

- QCDOC at BNL
- commodity clusters at Fermi Lab and Jefferson Lab
- $\sim 3 \times 10$ Teraflops distributed computing facility

QCDOC

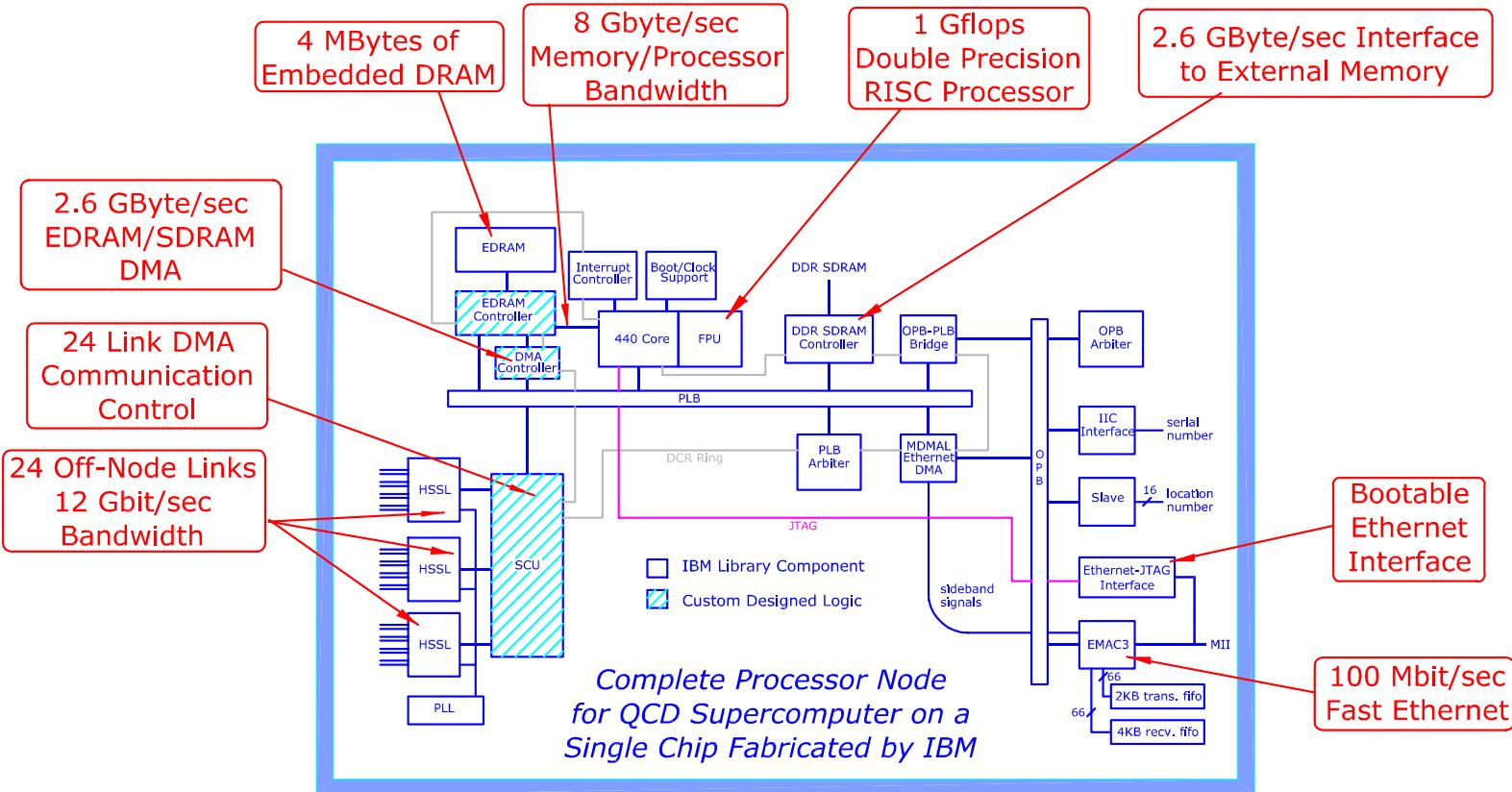
- next generation after QCDSP
- designed by Columbia University with IBM
- on design path to IBM Blue Gene
- Power PC nodes connected in a 6 dimensional torus
- processor/memory/communication on a single chip

Current RIKEN QCDSP machine

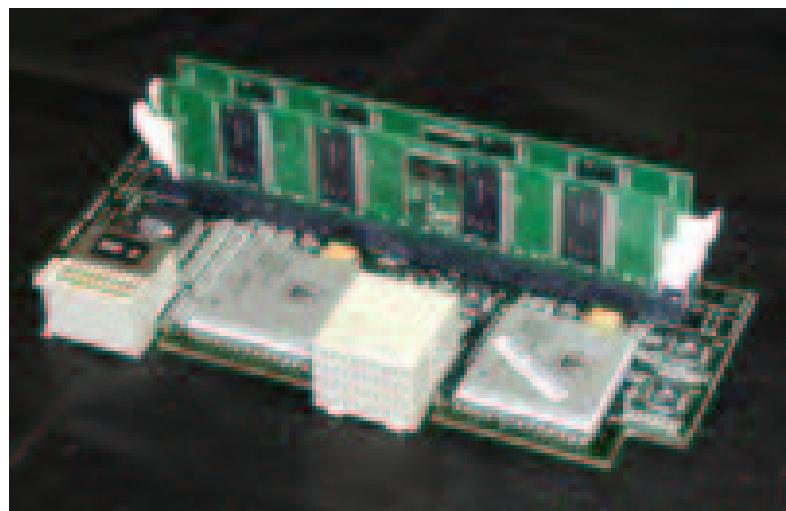


QCDOC places entire node on a single custom chip

QCDOC ASIC DESIGN



Mission-critical, custom logic (hatched) for high-performance memory access and fast, low-latency off-node communications is combined with standards-based, highly integrated commercial library components.



Schedule

- chip design: done --- first chips delivered beginning of June
- 64 node prototype now running
- 128 node prototype at Columbia: being completed
- 5 teraflop sustained RIKEN and UKQCD machines, 2004
- 10 teraflop sustained community machine: 2004 -- funding?
- 5-8 teraflop clusters at JLAB and FNAL: end of 2005

DOE panel review, Feb. 2003

Frank Wilczek (MIT) - chair

Roy Briere (CMU)

David Ceperley (NCSA-UIUC)

Candy Culhane (NSA)

Lynn Kissel (LLNL)

Michael Ogilvie (Washington Univ)

Robert Swendsen (CMU)

Peter Varman (NSF)

“In short, we feel the scientific merit of suggested program is very clearly outstanding.”

My Pet Problems

Chiral gauge theories

- parity conserving theories in good shape
- chiral theories (neutrinos) remain enigmatic
- non-perturbative definition of the weak interactions?
- related problem: supersymmetry

Fermion algorithms

- very awkward
- background “sign problem” unsolved
- why treat fermions and bosons so differently?

We need new ideas!